Magnetic Field Evaluation Methods for Measurement and Imaging Based on Nuclear Magnetic Resonance

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Abstract

A novel approach is proposed for magnetic field distribution testing, its symmetry, non-homogeneities and gradient detection and for non-linear area shapes indications. We have used Bin-Counts-Statistics (BCS) for listing the number of elements in data that lie in selected bins with increment of dx and 2D Fast Discrete Wavelet Transform (DWT) procedures. A radio-frequency (RF) narrow-gap planar coil system where the width-to-plane separation ratio (w/h) is more than 10:1 with limited dimensions of metal sheets was used as a model for magnetic field inhomogeneities evaluation. This coil is used for planar imaging using nuclear magnetic resonance methods. Small magnetic field homogeneity differences of the bi-planar RF coil show significant changes in BCS interpretation and also in selected wavelet components. The method seems to be useful both for new coil systems design and also for testing the magnetic fields: RF, stationary or gradient, used in NMR imaging and/or spectroscopy.

Keywords: magnetic field homogeneity, statistics, wavelet transform, nuclear magnetic resonance

1. Introduction

A high homogeneity of stationary and radio frequency magnetic fields for NMR measurement and imaging is desirable. By means of magnetic field calculation it is possible to plot the magnetic field distribution and evaluate its homogeneity according to known procedures (relative deviation, mean quadratic deviation, percentage deviation, 2D Fourier transform, etc.). The mathematical statistics offers Bin-Counts-Statistics methods [1], similar to histogram, expressed as continuous frequency curve, providing new, interesting view on the field inhomogeneities. With wavelets, one can perform multiresolution analysis, literally sorting signal components by their location and resolution scale [2], [3].

Magnetic field data represented by Bin-Counts-Statistics methods (BCS) and 2D Fast Discrete Wavelet Transform (DWT) shows new feature suitable eg for a new coil systems design. As an example a bi-planar RF coil designed in a form of metal planes was used.

2. Method

2.1 Planar Coil System

An RF narrow-gap planar coil system where the width-to-plane separation ratio (w/h) is more than 10:1 with limited dimensions of metal sheets was used as a model for magnetic field inhomogeneities evaluation, Fig.1.
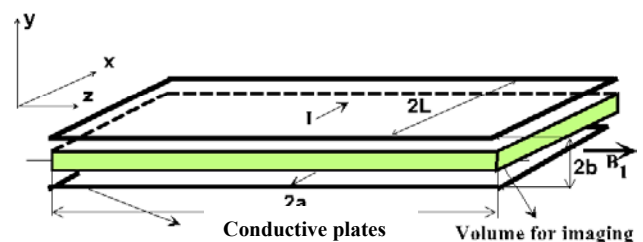

Fig.1 Central part of a planar RF coil system with limited plane length 2L, width w=2a, separated by a distance h=2b.

For the magnetic field $B_1 = H_z$ of the planar system the following formula was derived [6]:

$$H_z(x,y,z) = \frac{I}{\pi} \int_{-a}^{a} W_{in} V_{in} \, da \qquad (1)$$

where:

$$V_{in} = \frac{b_i - y}{(a - z)^2 + (b_i - y)^2}$$

$$W_{in} = Sin\left[ArcTan\left(\frac{L - x}{a - z}\right)\right] + Sin\left[ArcTan\left(\frac{L + x}{a - z}\right)\right]$$

Resultant magnetic field is a sum of two integrals, one for upper plane ($b_i = b$), the second one for the lower plane ($b_i = -b$). The feeding currents of the upper and lower planes are oriented in opposite directions, +I and –I.

The generated magnetic field in a rectangular volume was expressed as Percentage Field Deviation (PFD) with respect to the coil`s centre value of magnetic field:

$$PFD = \frac{H_z(x,y,z) - H(0,0,0)}{H(0,0,0)} 100 \quad [\%] \qquad (2)$$
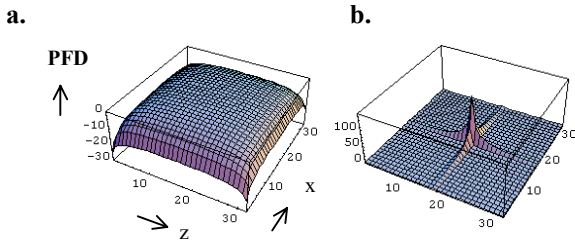
a.   **b.**

Fig.2 a. Percentage Field Deviation in the centre of the coil system, on z-axis. b. 2D Fourier Transform (absolute values) of the PFD data.

For design of the magnetic field distribution of the planar coil system the following requirements must be assumed:

- Maximal intensity of the magnetic field strength B_1 in the volume for imaging.

- Minimal inhomogeneities in the volume for imaging in a narrow gap.

The following optimization criteria for coil design should be taken into consideration: minimal amplitudes of the mean square deviation function or norm of the alternating part of the magnetic field distribution, minimal integral of the absolute values of this part. It is also possible to use a histogram of the field as a number of sampling points, which have the same field magnitude.

In our case for design and optimization of the planar coil system regarding minimal inhomogeneities of the generated magnetic field in a rectangular volume the Percentage Field Deviation (PFD) according to equation (2) was used.

2.2 Bin-Counts-Statistics

In the first step the Bin-Counts-Statistics for listing the number of elements in data that lie in selected bins with increment of dx (2-D BCS algorithm) was used [1].

As input data we have two square matrices for left and for right half of the coil system W1 and W2, see Fig.3. By the transposition of [W1, W2] we settle a direction of the BCS calculation and a shape of the resultant graphical representation [V1, V2]. For to get a symmetrical graphical result we need to reverse the output matrix V2. By joining the output matrices we can draw a ListPlot3D or ListContourPlot of the resultant data of the BCS procedure. Results are depicted in Fig.4.
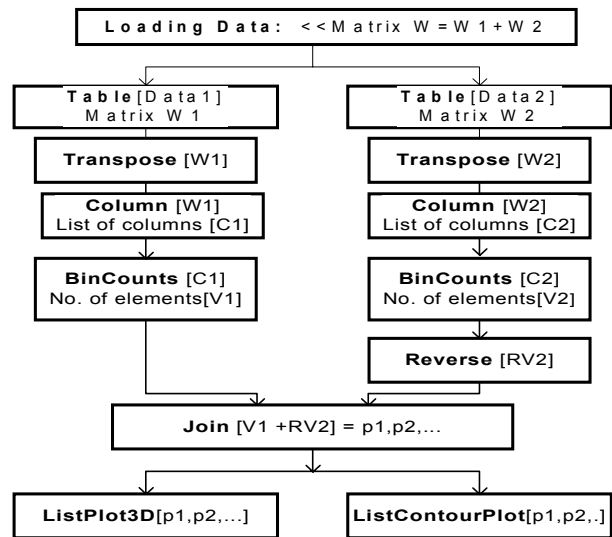

Fig.3 Flow chart of the BCS calculation. Dimension of the matrix is nc= 30 x 30, or nc = 40 x 40. BinCounts was calculated for every column, reverse and join data.

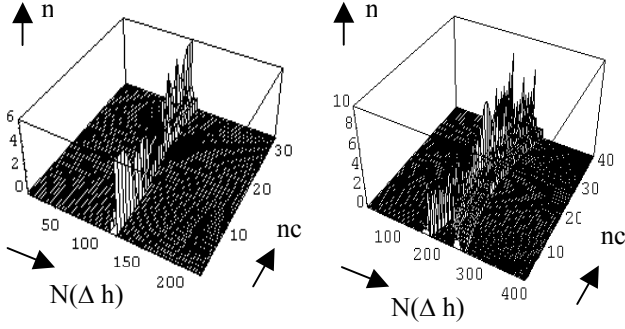*Fig.4   3D Plot of the BCS (for y = 0, field in between coils). n - number of elements,   N(Δh) - number of steps (selected bins). Left: quasi-homogeneous magnetic field, right: non-homogeneous magnetic field of the coil.*

2.3 Wavelet Transform

In the next step we want to show a possibility to use Wavelet Transform (WT) for interpretation of the magnetic field distribution.

With wavelets, one can perform multiresolution analysis, literally sorting signal components by their location and resolution scale [2]. Whereas Fourier Transform methods sort signals into their spectra, the wavelet transforms sort signal or data details into a locale-scale collection. Wavelets already enjoy connection with many fields and nuclear magnetic resonance imaging is starting to use them for signal and image processing [3], [4]. Magnetic field data represented by 2D Fast DWT shows new feature suitable eg for a new coil systems design. Small magnetic field homogeneity differences of the bi-planar RF coil show significant changes in selected wavelet components.

We have used two-dimensional scaling function or wavelet as a product of two one-dimensional functions:

$$\phi(x, y) = \phi(x)\phi(y) \tag{3}$$

and the dilation equation assumes the form

$$\phi(x, y) = 2\sum_{k,l} h_{k,l}\phi(2x - k, 2y - l). \tag{4}$$

We assume that both $\phi(x)$ and $\phi(y)$ satisfy the dilation equation:

$$\phi(x) = \sqrt{2}\sum_{k} h_k \phi(2x - k) \tag{5}$$

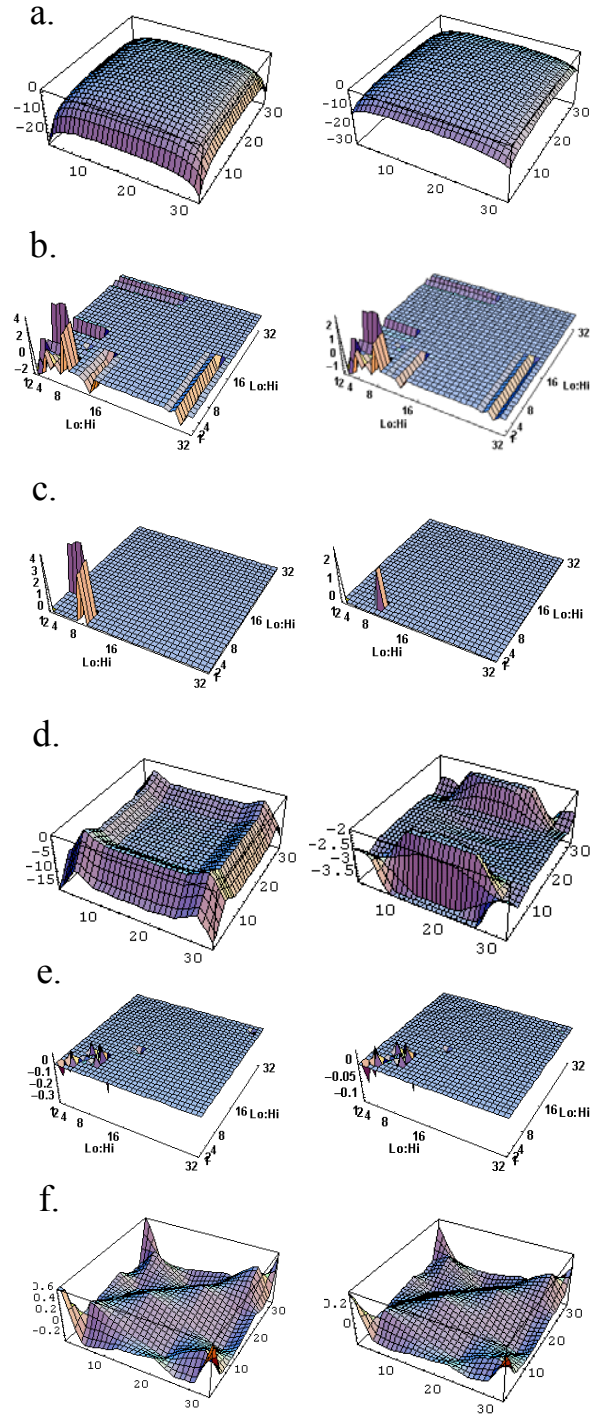for $h_{k,l} = h_k h_l$

Fig.5. Magnetic field evaluation using the 4th order Coifman wavelet system "Coif4". Left column: Non-homogeneous magnetic field. Right column: Quasi-homogeneous magnetic field.   Procedures:
a.   3D plot of the Percentage Field Deviation = [data]
b.   Trans2D[data, Coif4] =[trans]
c.   Threshold2D[trans,2]=[thresh]
d.   InverseTrans 2D[thresh, Coif4]
e.   Zero2DComponents[trans,c1,c2, c3,c4] =[Zero]
f.   Inverse Transform2D[Zero]

It means that the two-dimensional dilation equation is a product of two one-dimensional dilation equations.

In our case the 4th order Coifman wavelet system "Coif4" [5] as a low-pass filter representing the scaling function was applied.

For magnetic field evaluation the following DWT procedures were used: *trans=Transform2D[data, Coif4]*, where *[data] = Percentage Field Deviation*, Fig.5a,b.

The following performed procedures show significant amplitudes changes of low level wavelet components:

Threshold2D[trans, threshold 2]=[thresh], Fig.5c. Amplitudes of low level components for non-homogeneous magnetic field are increasing (in our example for about 100 %).

This procedure for lower values of threshold levels shows substantial changes in the after-filtered *InverseTrans2D[thresh,Coif4]*, see Figure 5d.

We get similar results by *Zero2DComponents[trans, c1,c2,c3,c4]=[Zero]*, Fig.5e, procedure deselecting lower levels components [c1,c2,c3]. After *Inverse Transform2D[Zero]* represented as a 3DPlot, the magnetic field components belonging to the selected wavelet components are seen, Fig.5f.

3. Conclusion

The Bin-Counts-Statistics interpretation method introduces a novel view on magnetic field distribution, its symmetry, gradient detection and indicates the non-linear area shapes.

An attempt was made to use 2D Fast Discrete Wavelet Transform procedures for magnetic field of RF coils evaluation regarding their non-homogeneities. The proposed methods seems to be useful both for new coil systems design and also for testing the magnetic fields: RF, stationary or gradient, used in NMR imaging and/or spectroscopy. New resultant features of the presented magnetic field representation could be also a suitable tool for magnetic field correcting system design.

For practical application in nuclear magnetic resonance imaging, a bi-planar RF coil was designed as a multiwire narrow-gap coil system instead of parallel conductive plates. For computation of such a coil system the formula (1) have been used, where integration was replaced by sum. The resultant magnetic field was created as a superposition of magnetic fields of all struts of the multiwire system. The final homogeneity of the magnetic field was given by number of coils and by strut distribution.

More coils systems and their magnetic fields used in nuclear magnetic resonance imaging were tested by the Bin-Counts-Statistics methods and by 2D Fast Discrete Wavelet Transform.

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