Measuring and Optimisation Methods for Basic Parameters Design in Physical Experiments

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ABSTRACT

The scope of this paper is to demonstrate a least square method for optimisation of basic parameters for selected physical experiment design where large input parameters adjustment is needed. The speed of calculation and experimentally verified results are promising to use this method by many physical projects.

We have demonstrated this method for computation of feeding currents of correcting coils for stationary magnetic field used in NMR imaging. The method needs to perform a magnetic field measurement in selected points of an assigned volume twice: when shim coils are switched off and afterward the measurement of magnetic field changes caused by switching on the feeding current of particular shim coil in each of selected points. A set of linear equations definition, determination of a target function and optimisation computations are procedures that provide optimal values of currents for shim coils. The proposed method because of its simplicity and speed of computation is convenient for basic adjustment of the magnetic field homogeneity by first magnet installation. It is also suitable for periodic testing and magnet inhomogeneities correction for MRI magnets especially in the case when the magnetic properties of the magnet surroundings are changed.

Keywords: measurement, optimisation, least square method, magnetic field, homogeneity, nuclear magnetic resonance.

1. INTRODUCTION

Using of the least square method for optimisation of basic parameters for a physical experiment is possible supposing the following conditions:

1. A direct physical coupling exists between adjustable input parameters and the intrinsic experiment i.e. there is a linear combination of input parameters and initial values of physical parameters in selected points.
2. Number of unknown parameters p is smaller (max. equal) as number of parameters measured in the experiment.
3. It is possible to measure (or calculate) with an adequate accuracy the parameters of the experiment in points that should be optimised.
4. It is possible to measure (or calculate) contributions (effects) of changes of known input parameters on resultant parameters of the experiment.
5. Physical limits of adjusted input parameters of actual experimental equipment are known.

The basic sequence of the solution is evident from Fig.1.

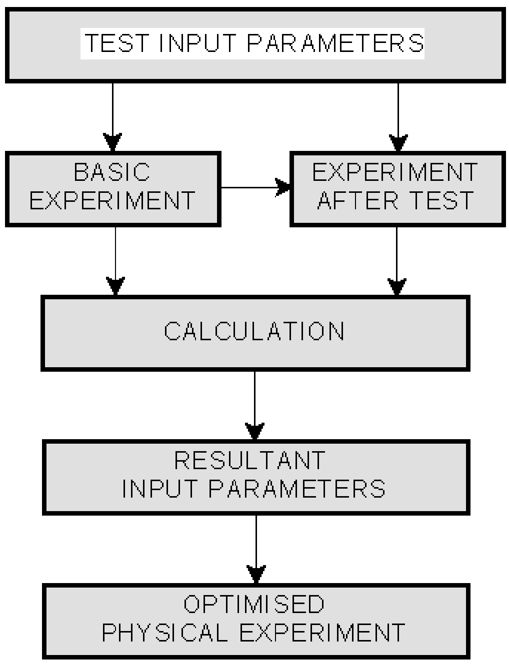

Fig.1 *Flowchart of the main optimisation procedure.*

The flowchart is indicating the following procedure:

- measurement (or calculation) of basic (initial) parameters of the experiment,
- measurement (or calculation) of contributions to unit changes of test input parameters,
- the computing algorithm is generating the resultant input parameters that after application to the initial experiment create an optimised physical experiment.

Next we describe a detailed procedure in an application to the concrete physical experiment

Magnetic field correction

Methods based on NMR principles (imaging and spectroscopy) need a source of stationary magnetic field with minimal inhomogeneities. No magnet generates an ideal homogeneous magnetic field and therefore a complicated coil system construction fed by separate currents power supplies for compensation (shimming) of particular magnetic inhomogeneous components (x, y, xy, yz, xz, x^2-y^2, xz^2, yz^2, z, z^2, z^3 z^4, ...) is needed [1]. For individual shim current setting one needs a complex mathematical computation performed by the magnet producer [2]. Final shim currents are set directly on site after real magnetic field inhomogeneities measurements (inhomogeneities caused by ferromagnetic objects placed near the magnet) using NMR magnetometers or directly by imaging [3].

An open question is the magnet inhomogeneity testing during its operation especially in situations when ferromagnetic objects distribution near the magnet is changed and when the magnet is not equipped with magnetic shielding.

Several methods for magnetic field correction and shim coils current calculation have been developed generally based on the spherical harmonic expansions and their derivatives. By computation of the coefficients for every component of the expansion using minimisation methods [1, 2, 4] or least squares method [5] it is possible to correct the magnetic field to achieve a high homogeneity.

In the paper we have designed new, simple and fast method for shim coil currents computation based on magnetic field values measured without shim coils (currents for shim coils are switched off) and magnetic field values after switching on the shim coils testing currents. No complicated expansions are needed. The testing currents can be adjusted generally to any value supposing that the change of magnetic field is measurable with acceptable precision. Naturally, for to do the computation effective we select equal testing currents for all shim coils (e.g. 1 A) or we group the shim coils with equal testing currents (e.g. 1, 5, 10 A).

2. METHOD

One of the main conditions of the designed method is the selection of points in the centre of the magnet in which we want adjust the required homogeneity of magnetic field in a real range. Obviously we measure and adjust magnetic field on a cylinder surface and in its inside, Fig. 2. We select the measuring points on three or five circular lines (or discs) symmetrical to the magnet centre with equal measuring point distribution on each circle plus points in the circles centres. In selected points the initial field and the field contributions of all shim coils were measured.

In the case of higher claims on homogeneity two or more measuring cylinders symmetrically to the magnet centre can be defined.

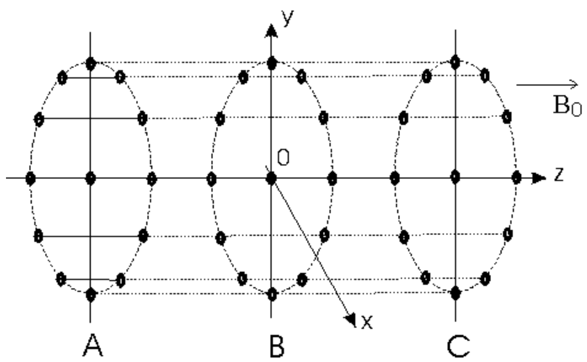

Fig.2 *Measuring point distribution example in three planes perpendicular to horizontal axes of an electromagnet.*

Theory

Our task belongs to domains of unknown parameters estimation in a linear regression function in the mathematical statistics. It is a problem where measurement of real values is performed (in our case magnetic field parameters $b_1, b_2, ..., b_n$) where every value is expressed as a linear combination of unknown parameters $I_1, I_2, ..., I_p$, (shim currents).

The determining equation for our task is as follows:

$$b_i = g_{1i} I_1 + g_{2i} I_2 + ... + g_{pi} I_p \qquad (1)$$

for $i = 1,2,....,n$

where

$g_{1i}, g_{2i}, \ldots\ldots, g_{pi}$ are known values of magnetic field differences corresponding to known testing currents of particular shim coils.

Provided that $p \leq n$ a matrix $\left\| g_{ji} \right\|$ where coefficients $g_{1i}, g_{2i}, \ldots\ldots, g_{pi}$ in equation (1) are in i-th column can be written in the form:

$$\mathbf{G} = \begin{Vmatrix} g_{11}, g_{12}, \ldots\ldots, g_{1n} \\ g_{21}, g_{22}, \ldots\ldots, g_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ g_{p1}, g_{p2}, \ldots\ldots, g_{pn} \end{Vmatrix} \quad (2)$$

For the following consideration we will suppose that rows of the matrix **G** are linearly independent.

It is known from the least square theory that values $I_1, I_2, \ldots., I_p$ are estimated as a minimum of the sum:

$$\mathrm{S} = \sum_{i=1}^{n} \left(b_i - g_{1i} I_1^* - g_{2i} I_2^* - \ldots\ldots - g_{pi} I_p^* \right)^2 \quad (3)$$

Than estimates $I_1^*, I_2^*, \ldots., I_p^*$ are determined from the equation:

$$\frac{\partial S}{\partial I_h} = 0 \quad (4)$$

For h = 1, 2, ..., p.

In the mathematical statistics the equation (4) is named as normal equation.

Let as consider that

$$a_{hj} = \sum_{i=1}^{n} g_{hi} g_{ji} \qquad \text{(h, j = 1,2,…,p)} \qquad (5)$$

$$a_h = \sum_{i=1}^{n} g_{hi} b_i \qquad \text{(h, = 1,2,…,p)} \qquad (6)$$

we have

$$\partial S / \partial I_h^* = -2(a_h - a_{h1} b_1^* - a_{h2} b_2^* - \ldots - a_{hp} b_p^*) .$$

The normal equations are now in the form:

$$\begin{aligned} & a_{11} I_1^* + a_{12} I_2^* + \ldots + a_{1p} I_p^* = a_1 \\ & a_{21} I_1^* + a_{22} I_2^* + \ldots + a_{2p} I_p^* = a_2 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & a_{p1} I_1^* + a_{p2} I_2^* + \ldots + a_{pp} I_p^* = a_p \end{aligned} \qquad (7)$$

Let us indicate:

$$\mathbf{A} = \left\| a_{hj} \right\|, \quad \mathbf{I}^* = \begin{Vmatrix} I_1^* \\ . \\ . \\ I_p^* \end{Vmatrix}, \quad \mathbf{a} = \begin{Vmatrix} a_1 \\ . \\ . \\ a_p \end{Vmatrix}$$

Then the equation (7) has the form:

$$\mathbf{A I}^* = \mathbf{a} \qquad (8)$$

According to (5) we can write:

$$\mathbf{A} = \mathbf{G G}^{`}, \qquad (9)$$

where $\mathbf{G}^{`}$ is a transpose matrix to the matrix $\mathbf{G}$.

matrix $\mathbf{A}$ is a symmetrical matrix type "p x p" depending on known coefficients g_{ji} of the determining equation (1).

Estimates of $\mathbf{I}^*$ are exactly defined by term:

$$\mathbf{I}^* = \mathbf{A}^{-1} \mathbf{a} \ . \qquad (10)$$

From (6) results:

$$\mathbf{a} = \mathbf{G b} \ , \qquad (11)$$

where $\mathbf{b} = \begin{Vmatrix} b_1 \\ . \\ . \\ b_n \end{Vmatrix}$.

Using (10) and (11) we get the solution of the equation (4) in a matrix form:

$$\mathbf{I} = [\mathbf{G.G}^{`}]^{-1} \mathbf{.G.b} \qquad (12)$$

where $[\mathbf{G.G}^{`}]^{-1}$ is an inversion matrix to the matrix $\mathbf{G.G}^{`}$, where

$\mathbf{b} = (b_{1,} b_2 \ldots\ldots. b_n)^{`}$ is a vector of initial magnetic field

$\mathbf{I} = (I_1^*, I_2^*, \ldots., I_p^*)^{`}$ is a vector of calculated current values of shim coils.

Simple solution of our problem is given by equation:

$$\mathbf{I.G} = \mathbf{b} \qquad (13)$$

where $\mathbf{b} + \mathbf{b}_r = 0$, and $\mathbf{b_r}$ is the real magnetic field without shimming.

It is necessary to remark that $\mathbf{b_r}$ and $\mathbf{b}$ in a practical calculation represent magnetic field values after homogeneous component (in a selected range) subtraction. Using shim coils we are able to correct only differences of the magnetic field from its mean value in a selected interval. Computing of the shim coils current according to equations (12) or (13) is exact. But in praxis this method does not always satisfy because the calculated values of shim currents can be higher than power supply possibilities.

Computing algorithm

In the designed algorithm we are looking for such correcting current values that minimise the magnetic field inhomogeneity respecting technical parameters of the equipment. Naturally in this case it is not possible to use the equation $\mathbf{I}.\mathbf{G} = \mathbf{b}$. It is necessary to find a criterion for the final quality of the magnetic field. The mathematical statistics proposes the following measures of dispersion: width of the span, mean value, mean square deviation. In the described method we have used a minimisation procedure of the mean square deviation of the magnetic field in selected points. The designed algorithm calculates with limited currents in a real range. Computing program is using functions like Min(S) or FindMinimum(S) where S is given by equation (3).

For our task we can use the following sequence:

1. Measurement of the magnetic field b_i in all points of selected planes, join data
2. Mean value: $(\sum b_i)/n = b_m$, Oscillating component: $b_a = b_i - b_m$
3. Primary inhomogeneities estimation: $b_{in\hom} = Max[b_i] - Min[b_i]$
4. Measurement of magnetic field contributions of every correction coil in all selected points: $[g_{1i}, g_{2i}, \ldots\ldots, g_{pi}]$
5. Basic equation construction
$$S = \sum_{i=1}^{n} \left(b_i - g_{1i} I_1^* - g_{2i} I_2^* - \ldots\ldots - g_{pi} I_p^*\right)^2$$
6. Find minimum of the target function S
7. Output $\Rightarrow I_1^*, I_2^*, \ldots., I_p^*$
8. Test using equation (3).

The calculated values can be verified by adjustment of calculated currents for every shim coil and by experimental measurement of the resultant magnetic field.

Experimental results

We have used the designed method for magnetic field correction of a home-made whole-body NMR imager 0.1 Tesla. The initial field and the testing current contributions for all shim coils were measured (using NMR magnetometer Bruker) in 3 planes on circles and their centres, together 3 x 13 points, Fig. 2. For measuring probe positioning a mask was used, Fig. 3.

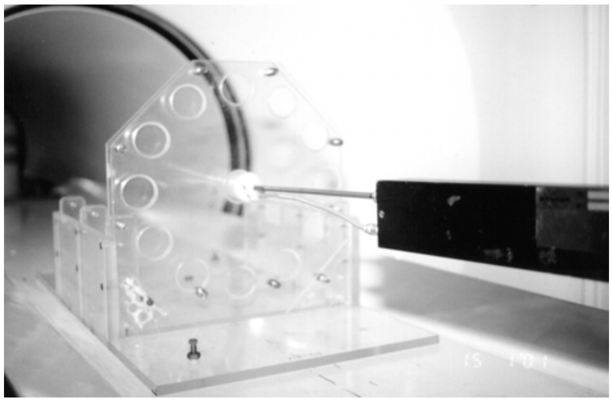

Fig. 3. *Measuring mask with bushings for NMR magnetometer probe*

In the selected point the initial field and the field contributions of all shim coils were measured in fast sequence to avoid the possible time instability of the basic magnetic field. For extreme accuracy (uncertainty lower than 1 ppm) a special NMR stabiliser of the basic magnetic field must be used. The stabiliser is switched on only during the time when no shim coil is energised.

On measured data the mean square algorithm using function FindMinimum(S), [6], was applied. The calculation was repeated several times changing the starting value for minimum search. The resultant values $I_1^*, I_2^*, \ldots., I_p^*$ were tested substituting to the equation (3) and depicted graphically in Fig. 4.

In the case when some of the calculated current was higher as maximal value of the power supply, the maximal value was substituted to the eqn. (3) as a constant and the calculation was repeated. After adjusting of the resultant shim currents for every coil a final magnetic field measurement was performed. Our results showed excellent correspondence of calculated and measured values.

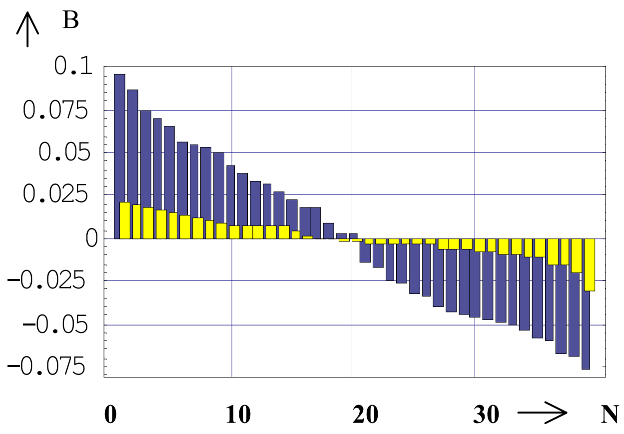

Fig. 4. *Bar graph of sorted relative values of the magnetic field B without correction (high bars) and with energised shim coils (small bars) fed by calculated currents. N is relative position number.*

3. CONCLUSION

In the paper we have demonstrated a least square method for optimisation of basic parameters for selected physical experiment design where large input parameters adjustment is needed. We have shown how to use this method for stationary magnetic field homogeneity calculation. The homogeneous magnetic field is a basic condition for imaging based on nuclear magnetic resonance.

Determining equation (1) was used either for exact solution given by eqn. (10) or (12) or for a minimum search algorithm and procedure for the function (3) based on iterations. Exact procedures in some cases produced unreal high values of correcting currents. Iteration method returns very fast successful results under reasonable physical conditions.

The designed method can be used for regular testing of a basic electromagnet and for shim coil currents adjusting with the goal to create optimal conditions of NMR experiments.

The speed of calculation and experimentally verified results are promising to use this method by many physical projects where large input parameters adjustment is needed.

Acknowledgements

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