Wavelet Transform used for Magnetic Field Evaluation for Imaging Based on Nuclear Magnetic Resonance

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Abstract

A novel approach is proposed for magnetic field distribution testing, its symmetry, non-homogeneities and gradient detection and for non-linear area shapes indications. We have used 2D Fast Discrete Wavelet Transform (DWT) procedures. A radio-frequency (RF) narrow-gap bi-planar coil system was used as a model. This coil is used for planar imaging using nuclear magnetic resonance methods. Small magnetic field homogeneity differences of this coil show significant changes in selected wavelet components. The method is useful both for new coil systems design and optimization and also for testing the magnetic fields.

1. Introduction

High homogeneity of stationary and radio frequency magnetic fields for NMR measurement and imaging is desirable. By means of magnetic field calculation it is possible to plot the magnetic field distribution and evaluate its homogeneity according to known procedures (relative deviation, mean quadratic deviation, percentage deviation, 2D Fourier transform, histograms, etc.). With wavelets, one can perform multiresolution analysis, literally sorting signal components by their location and resolution scale [1, 2].

Magnetic field data represented by 2D Fast Discrete Wavelet Transform shows new features suitable eg for a new coil systems design using genetic algorithms where an object function is represented by a selected WT component. As an example a bi-planar RF coil designed in a form of metal planes was used.

2. Method

An RF narrow-gap planar coil system (where the width-to-plane separation ratio (w/h) is more than 10:1 with limited dimensions of metal sheets) was used as a model for magnetic field inhomogeneities evaluation, Fig. 1.

For the magnetic field $B_1 = H_z$ of the planar system the following formula was derived:

$$H_z(x, y, z) = \frac{I}{\pi} \int_{-a}^{a} \frac{b_i - y}{(a - z)^2 + (b_i - y)^2} \left\{ Sin\left[ArcTan \frac{L - x}{a - z} \right] + Sin\left[ArcTan \frac{L + x}{a - z} \right] \right\} da \qquad (1)$$

Resultant magnetic field is a sum of two integrals, one for upper plane (b_i=b), the second one for the lower plane (b_i=-b). The feeding currents of the upper and lower planes are oriented in opposite directions, +I and –I.

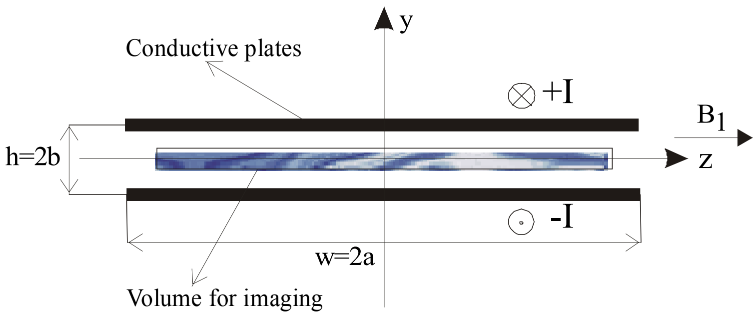

Fig.1 Central part of a planar RF coil system with limited plane length 2L, width w=2a, separated by a distance h=2b.

The generated magnetic field in a rectangular volume was expressed as Percentage Field Deviation (PFD) with respect to the coil`s centre value of magnetic field:

$$PFD = \frac{H_z(x,y,z) - H(0,0,0)}{H(0,0,0)} 100 \quad [\%] \tag{2}$$

For the planar coil system the following requirements must be assumed: Maximal intensity of the magnetic field strength B_1 in the volume for imaging and minimal inhomogeneities in the volume for imaging in a narrow gap.

In our case for design and optimization of the planar coil system regarding minimal inhomogeneities of the generated magnetic field in a rectangular volume the Percentage Field Deviation (PFD) according to equation (2) was used.

2.1 Wavelet Transform

Wavelets provide convenient sets of basis functions for function spaces used for sorting signal components by their location and resolution scale. Whereas Fourier Transform methods sort signals into their spectra, the wavelet transforms sort signal or data details into a locale-scale collection. Wavelets already enjoy connection with many fields and nuclear magnetic resonance imaging is starting to use them for signal and image processing [2, 3]. Magnetic field data represented by 2D Fast DWT shows new feature suitable eg for a new coil systems design. Small magnetic field homogeneity differences of the bi-planar RF coil show significant changes in selected wavelet components.

We have used two-dimensional scaling function or wavelet as a product of two one-dimensional functions:

$$\phi(x,y) = \phi(x)\phi(y) \tag{3}$$

and the dilation equation assumes the form

$$\phi(x,y) = 2\sum_{k,l} h_{k,l}\phi(2x-k, 2y-l). \tag{4}$$

We assume that both $\phi(x)$ and $\phi(y)$ satisfy the dilation equation:

$$\phi(x) = \sqrt{2}\sum_{k} h_k \phi(2x-k) \qquad \text{for} \quad h_{k,l} = h_k h_l \tag{5}$$

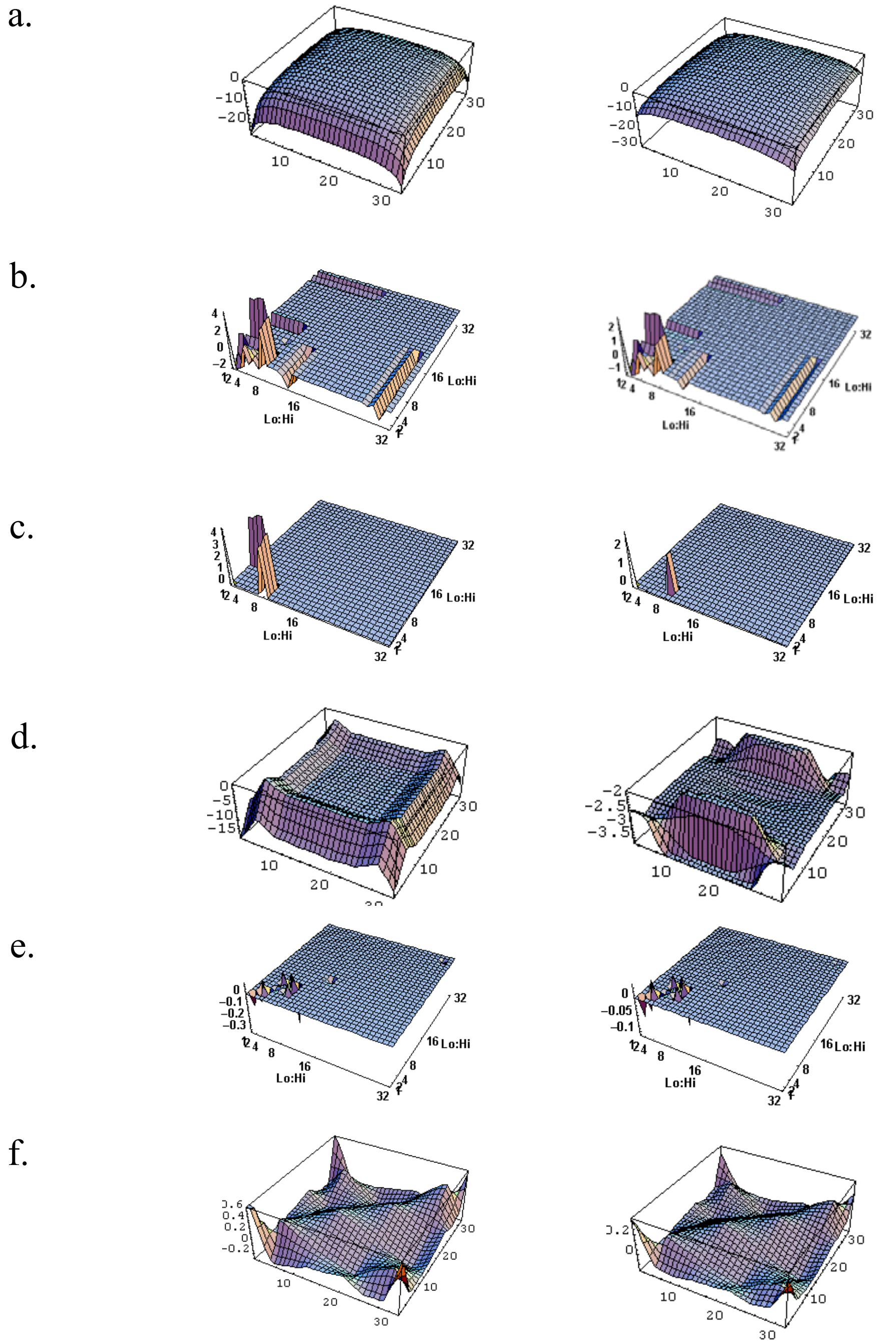

Fig. 2. Magnetic field evaluation using the 4th order Coifman wavelet system "Coif4". Left column: Non-homogeneous magnetic field. Right column: Quasi-homogeneous magnetic field. Procedures:

a. 3D plot of the Percentage Field Deviation = [data]
b. Trans2D[data, Coif4] =[trans]
c. Threshold2D[trans,2]=[thresh]
d. InverseTrans 2D[thresh, Coif4]
e. Zero2DComponents[trans,c1,c2, c3,c4] =[Zero]
f. Inverse Transform2D[Zero]

In our case the 4th order Coifman wavelet system “Coif4” [4] as a low-pass filter representing the scaling function was applied.

For magnetic field evaluation the following DWT procedures were used: *trans=Transform2D[data, Coif4],* where *[data] = Percentage Field Deviation*, Fig. 2a,b.

The following performed procedures show significant amplitudes changes of low level wavelet components:

Threshold2D[trans, threshold 2]=[thresh], Fig. 2c. Amplitudes of low level components for non-homogeneous magnetic field are increasing (in our example for about 100 %).

This procedure for lower values of threshold levels shows substantial changes in the after-filtered *InverseTrans2D [thresh,Coif4]*, see Fig. 2d.

We get similar results by *Zero2DComponents[trans, c1,c2,c3,c4]=[Zero],* Fig. 2e, procedure deselecting lower levels components [c1,c2,c3]. After *Inverse Transform-2D[Zero]* represented as a 3DPlot, the magnetic field components belonging to the selected wavelet components are seen, Fig. 2f.

3. Conclusion

An attempt was made using 2D Fast Discrete Wavelet Transform procedures for magnetic field of an RF coil evaluation regarding its non-homogeneities. The proposed method seems to be useful both for new coil systems design, optimisation and also for testing the magnetic fields: RF, stationary or gradient, used in NMR imaging and/or spectroscopy or as a suitable tool for magnetic field correcting system design.

In our case the Wavelet Transform has been used for magnetic field optimisation. A selected Wavelet Transform component (eg. *Threshold2D)* has been chosen as an “object function”. The maximal value of the object function represents the level of inhomogeneities. The task of an optimisation procedure (based on genetic algorithm) was to minimise the peak value of the object function (see Fig.2 c.). The advantage: significantly faster computer procedure in comparison with classical methods using relative deviations, Fourier transform, histograms, etc.

For practical application in nuclear magnetic resonance imaging, a bi-planar RF coil was designed as a multiwire narrow-gap coil system instead of parallel conductive plates.

More coils systems and their magnetic fields used in nuclear magnetic resonance imaging were tested by 2D Fast Discrete Wavelet Transform.

References

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